sets of values are $\begin{cases} c = 1 \\ a = 1 \end{cases}$, $\begin{cases} c = 2 \\ a = 1 \end{cases}$, $\begin{cases} c = 3 \\ a = 2 \text{ or } 4 \end{cases}$, etc. The first set is the one given in the problem. The second gives $m'/n' = (m + 4n)/(2m + n)$ which yields the series 1/1, 5/3, 17/13, 69/47, ...

(*b*) Almost the same work leads to similar results in the case of any surd, $\sqrt{k}$. We find $a = d$, $b = kc$, $2a \geqq c(k - 1)$, $kc^2 > a^2$. These possess, among others, the following solutions for the case where $k = 3$.

$$\begin{cases} c = 1 \\ a = 1 \end{cases}, \begin{cases} c = 2 \\ a = 3 \end{cases}, \begin{cases} c = 3 \\ a = 4 \text{ or } 5 \end{cases}, \text{ etc.}$$

The first set gives $m'/n' = (m + 3n)/(m + n)$, which yields the series 1/1, 4/2 = 2/1, 5/3, 14/8 = 7/4, 19/11, ...

Also solved by NORMAN ANNING.

GEOMETRY.

466. Proposed by HORACE OLSON, Chicago, Illinois.

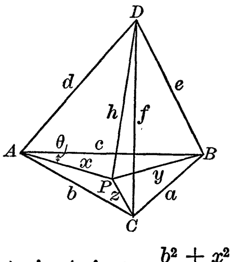
Given the edges of a triangular pyramid, find the radius of the inscribed sphere.

SOLUTION BY A. H. HOLMES, Brunswick, Maine.

Calling the radius of the sphere r, and using the notation in the figure, we have,

$$h^2 = d^2 - x^2 = c^2 - y^2 = f^2 - z^2,$$

$$\cos\theta = \frac{c^2 + x^2 - y^2}{2cx} = \frac{c^2 + d^2 - e^2}{2cx} = \frac{n}{x}. \qquad (x\cos\theta = n.)$$



$$\cos(A - \theta) = \cos A \cos\theta + \sin A \sin\theta = \frac{b^2 + x^2 - z^2}{2bx} = \frac{b^2 + d^2 - f^2}{2bx} = \frac{m}{x}.$$

Hence,

$$\sin\theta = \frac{m - n\cos A}{x \sin A}, \qquad x\sin\theta = \frac{m - n\cos A}{\sin A}.$$

Hence,

$$x^2 = n^2 + \frac{(m - n\cos A)^2}{\sin^2 A}, \qquad \text{and} \qquad h = \frac{\sqrt{(d^2 - n^2)\sin^2 A - (m - n\cos A)^2}}{\sin A}.$$

Hence,

$$3 \text{ times contents of pyramid} = \frac{bc}{2}\sqrt{(d^2 - n^2)\sin^2 A - (m - n\cos A)^2},$$

and therefore,

$$r = \frac{bc\sqrt{(d^2 - n^2)\sin^2 A - (m - n\cos A)^2}}{2(ABC + ACD + ABD + BCD)},$$

in which

$$m = \frac{b^2 + d^2 - f^2}{2b}, \qquad n = \frac{c^2 + d^2 - e^2}{2c}, \qquad \cos A = \frac{b^2 + c^2 - a^2}{2bc},$$

$$\sin A = \sqrt{1 - \frac{(b^2 + c^2 - a^2)^2}{4b^2c^2}}, \qquad ABC = \sqrt{s(s-a)(s-b)(s-c)},$$

and similarly for ABD, ACD and BCD.

Also solved by WALTER C. EELLS and J. W. CLAWSON.

CALCULUS.

380. Proposed by C. N. SCHMALL, New York City.

Show that

$$\int_0^\infty \left[\frac{1}{1^4 + x^2} + \frac{1}{2^4 + x^2} + \frac{1}{3^4 + x^2} + \cdots\right] dx = \frac{\pi^3}{12},$$

where the series in the brackets is infinite.

SOLUTION BY A. M. HARDING, University of Arkansas.

For all values of x in the interval $(0, \infty)$ the nth term of the given series has the property

$$\frac{1}{n^4 + x^2} \leqq \frac{1}{n^4}.$$

Now the series $\sum_{n=1}^{\infty} 1/n^4$ converges. Hence, the given series is uniformly convergent in the interval $(0, \infty)$.

Each term of the series is continuous in the interval $(0, \infty)$.

Hence, it may be integrated term by term. Hence,

$$\int_0^\infty \left[\frac{1}{1^4 + x^2} + \frac{1}{2^4 + x^2} + \frac{1}{3^4 + x^2} + \cdots\right] dx = \frac{\pi}{2}\left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right].$$

It can be easily shown that

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6}.$$

Hence, the given integral $= \dfrac{\pi^3}{12}$.

Also solved by S. A. JOFFE, and J. A. CAPARO.

299. Proposed by B. F. FINKEL, Drury College.

A cone rests in two fluids which do not mix, with its vertex downwards and its base in the surface of the upper fluid; to find how much its density must be increased that it may rest with its base in the common surface of the fluids. (From Walton's *Hydrostatical Problems.*)

SOLUTION BY J. F. BRACHO, University of Notre Dame.

Let, w = density of cone in first position, w_1 = density of cone in second position, d_1 = density of upper fluid, d_2 = density of lower fluid. We have:

$$DC = \frac{AB + OD}{OA} = \frac{r(h-a)}{h}.$$